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Advanced Nonlinear Control of Three Phase Shunt Active Power Filter

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Abstract— **The problem of controlling three-phase shunt active power filter (TPSAPF) is addressed in this paper in presence of nonlinear loads of rectifier type. The control objective is twofold: (i) compensation of harmonic currents and the reactive power absorbed by the nonlinear load; (ii) regulation the DC bus voltage of the converter. A nonlinear controller which has a cascade structure comprises two loops, an inner-loop which is formed by two non-linear regulators is designed, using the Backstepping technique, to provide the harmonic compensation and an outer-loop is designed to regulate the DC bus voltage of the converter. Controller performances are illustrated by numerical simulation in Matlab/Simulink environment.**

I. INTRODUCTION

Nowadays, with the wide use of nonlinear loads and electronic equipment in distribution systems, the problem of power quality (PQ) has become increasingly serious. This fact has lead to more stringent requirements regarding PQ which include the search for solutions for such problems [1].

In the case of harmonic pollution, the solution can consider the use of passive filters however these filters have the disadvantage of having a fixed compensation and can generate resonance problems. In this way, the active power filter (APF) appears as the best dynamic solution for harmonic compensation [2]. In concrete, active power filters are devices designed to improve the power quality in distribution networks. In order to reduce the injection of non-sinusoidal load currents, shunt active power filters can be connected in parallel with the nonlinear loads.

In recent years, three-phase four-wire shunt active power filters have appeared as an effective method to solve the problem caused by harmonic and unbalanced currents as well as to compensate load reactive power. In these filters, three topologies for current-controlled voltage source inverter are commonly used, namely the Four-Leg Full-Bridge (FLFB) topology, the Three-Leg Split-Capacitor (TLSC) topology and the four-leg split-capacitor (FLSC) topology. These topologies were presented in the early 90s [3], and the research work has been done on only those topologies. Indeed many publications on their control have appeared ever since [4]-[6].

In this paper, a control strategy is developed for other topology named: Three-phase shunt active power filter or Two-Leg Split-capacitor (Two LSC) in the presence of non-linear loads. A nonlinear controller that has a cascade structure has two nested loops, an inner loop which is formed by two non-linear regulators is designed, using the Backstepping technique, to ensure compensation of harmonic currents and reactive power absorbed by the non-linear load. The outer-loop is designed to regulate the DC bus voltage of the converter. This theoretical result is confirmed by numerical simulations.

The paper is organized as follows: The system includes the electric network and the DC/AC converter is modeled in section 2, the control problem is formulated in section 3 which also includes the design, the stability analysis in section 4. Performances controller are illustrated by simulation in section 5. A general conclusion ends the paper in the last section.

II. ACTIVE POWER FILTER MODELING

A. Active Power Filter Topology

A reduced-part three-phase shunt active power filter under study has the structure of Figure 1. It also has two-Leg less than full-bridge configuration. In the DC side it has two identical capacitors values of energy storage C_f. In the AC side the TPSAPF is connected in parallel with a nonlinear load to the electric network which is modeled by three sinusoidal AC voltage sources that form a directly balanced three-phase system. The circuit operates according to the well known Pulse Width Modulation principle (PWM),[7],[8],[9],[10]. Finally the supply network contains three voltages that form a direct balanced three-phase system is given by:

$$\begin{pmatrix} v_{n1}(t) \\ v_{n2}(t) \\ v_{n3}(t) \end{pmatrix} = E \begin{pmatrix} Sin(\omega_n t) \\ Sin\left(\omega_n t - \frac{2\pi}{3}\right) \\ Sin\left(\omega_n t - \frac{4\pi}{3}\right) \end{pmatrix} \tag{1}$$

where E and ω_n denote respectively the amplitude and the frequency of system ($v_{n1}(t)$, $v_{n2}(t)$, $v_{n3}(t)$).The resulting current in three-phase load is given by its Fourier expansion:

$$\begin{pmatrix} i_{L1}(t) \\ i_{L2}(t) \\ i_{L3}(t) \end{pmatrix} = \sum_{h=1}^{\infty} I_h \begin{pmatrix} Sin(h\omega_n t + \varphi_h) \\ Sin\left(h\omega_n t + \varphi_h - \frac{2\pi}{3}\right) \\ Sin\left(h\omega_n t + \varphi_h - \frac{4\pi}{3}\right) \end{pmatrix} \tag{2}$$

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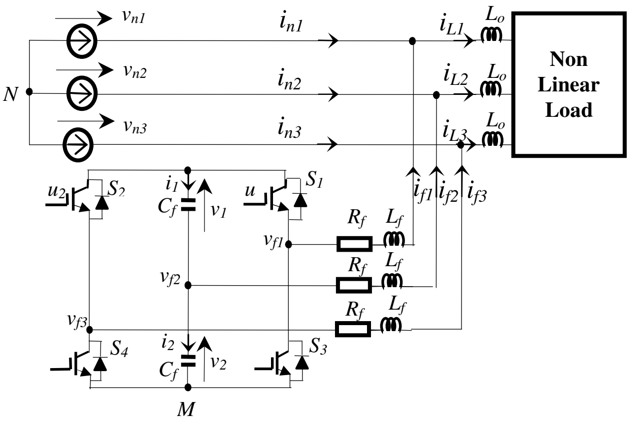

Figure 1. Three-phase shunt APF

B. Active Power Filter Modeling

The modeling of the three-phase shunt APF is performed by applying the usual Kirchhoff's laws. Doing so, one easily gets:

$$L_f \frac{di_{f1}}{dt} + R_f i_{f1} = v_{f1} - v_{n1} \tag{3a}$$

$$L_f \frac{di_{f2}}{dt} + R_f i_{f2} = v_{f2} - v_{n2} \tag{3b}$$

$$L_f \frac{di_{f3}}{dt} + R_f i_{f3} = v_{f3} - v_{n3} \tag{3c}$$

$$C_f \frac{dv_1}{dt} = i_1 \tag{3d}$$

$$C_f \frac{dv_2}{dt} = i_2 \tag{3e}$$

The three-phase shunt active power filter undergoes the equations:

$$v_{f1M} = \left(\frac{1+\mu_1}{2}\right)(v_1+v_2) \tag{4a}$$

$$v_{f2M} = v_2 \tag{4b}$$

$$v_{f3M} = \left(\frac{1+\mu_2}{2}\right)(v_1+v_2) \tag{4c}$$

$$i_1 = -\left(\frac{1+\mu_1}{2}\right)i_{f1} - \left(\frac{1+\mu_2}{2}\right)i_{f3} \tag{4d}$$

$$i_2 = \left(\frac{1-\mu_1}{2}\right)i_{f1} + \left(\frac{1-\mu_2}{2}\right)i_{f3} \tag{4e}$$

The switching function μ_k of the inverter is defined by:

$$\mu_k = \begin{cases} +1 & If \quad S_k \quad is \quad ON \quad and \quad S_{k+2} \quad is \quad OFF \\ -1 & If \quad S_k \quad is \quad OFF \quad and \quad S_{k+2} \quad is \quad ON \end{cases}$$

$$k \in \{1,2\}$$

$$v_{f1} = v_{f1M} + v_{MN} \tag{5a}$$

$$v_{f2} = v_{f2M} + v_{MN} \tag{5b}$$

$$v_{f3} = v_{f3M} + v_{MN} \tag{5c}$$

as (v_{f1}, v_{f2}, v_{f3}) is a symmetric system then

$$v_{f1} + v_{f2} + v_{f3} = 0 \tag{5d}$$

$$v_{MN} = -\frac{1}{3}\left(v_o + v_2 + \left(\frac{\mu_1+\mu_2}{2}\right)v_o\right) \tag{5e}$$

$$v_{f1} = \left(\frac{2\mu_1-\mu_2}{6}\right)v_o + \frac{v_d}{6} \tag{5f}$$

$$v_{f2} = \left(\frac{\mu_1+\mu_2}{6}\right)v_o - \frac{v_d}{3} \tag{5g}$$

$$v_{f3} = \left(\frac{2\mu_2-\mu_1}{6}\right)v_o + \frac{v_d}{6} \tag{5h}$$

with $v_o = v_1 + v_2$ and $v_d = v_1 - v_2$

Substituting (4d-e) and (5f-h) in (3) one obtains the instantaneous model of the filter:

$$\frac{di_{f1}}{dt} = -\frac{R_f}{L_f}i_{f1} + \left(\frac{2\mu_1-\mu_2}{6L_f}\right)v_o + \frac{v_d}{6L_f} - \frac{v_{n1}}{L_f} \tag{6a}$$

$$\frac{di_{f2}}{dt} = -\frac{R_f}{L_f}i_{f2} - \left(\frac{\mu_1+\mu_2}{6L_f}\right)v_o - \frac{v_d}{3L_f} - \frac{v_{n2}}{L_f} \tag{6b}$$

$$\frac{di_{f3}}{dt} = -\frac{R_f}{L_f}i_{f3} + \left(\frac{2\mu_2-\mu_1}{6L_f}\right)v_o + \frac{v_d}{6L_f} - \frac{v_{n3}}{L_f} \tag{6c}$$

$$\frac{dv_o}{dt} = -\left(\frac{\mu_1 i_{f1} + \mu_2 i_{f3}}{C_f}\right) \tag{6d}$$

$$\frac{dv_d}{dt} = -\left(\frac{i_{f1} + i_{f3}}{C_f}\right) \tag{6e}$$

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The binary control signals μ_1, μ_2 are produced by a PWM generator and take its values in the finite set {-1, 1}, the switched converter model (6) is a variable structure system and therefore inadequate for the design of a continuous control laws. To overcome this difficulty we resorted to using the averaged model where the different state variables and control laws are replaced by their average values over cutting intervals [11]. Since equation (6b) is a linear combination of the equations (6a) and (6c) then the obtained averaged model of the three-phase shunt APF is the following:

$$\dot{x}_1 = -\frac{R_f}{L_f}x_1 + \frac{U_1}{L_f}x_3 + \frac{1}{6L_f}x_4 - \frac{v_{n1}}{L_f} \tag{7a}$$

$$\dot{x}_2 = -\frac{R_f}{L_f}x_2 + \frac{U_2}{L_f}x_3 + \frac{1}{6L_f}x_4 - \frac{v_{n3}}{L_f} \tag{7b}$$

$$\dot{x}_3 = -\left(\frac{u_1 x_1 + u_2 x_2}{C_f}\right) \tag{7c}$$

$$\dot{x}_4 = -\left(\frac{x_1 + x_2}{C_f}\right) \tag{7d}$$

with $U_1 = \left(\frac{2u_1 - u_2}{6}\right)$ and $U_2 = \left(\frac{2u_2 - u_1}{6}\right)$

where x_1, x_2, x_3, x_4, u_1 and u_2 respectively denote the average values over cutting periods of the signals i_{f1}, i_{f3}, v_o, v_d, μ_1 and μ_2. The system (7) is clearly nonlinear because of the product between the state variables x_1, x_2, x_3, x_4 and the control inputs u_1 and u_2. It is assumed that all signals are measurable.

III. CONTROLLER DESIGN

The control objective of the three-phase shunt APF is to compensate the harmonics and reactive power required by the nonlinear load. The controller synthesis is carried out by cascading two loops. First an inner current loop which is formed by two non-linear regulators is designed to compensate the reactive and harmonic power. Second an external loop is constructed to ensure control of the voltage in the DC side of TPSAPF. The controller has the structure of Figure 2. The currents i_{f1}, i_{f3} are controlled in the system and due to symmetry of the system i_{f2} is being controlled.

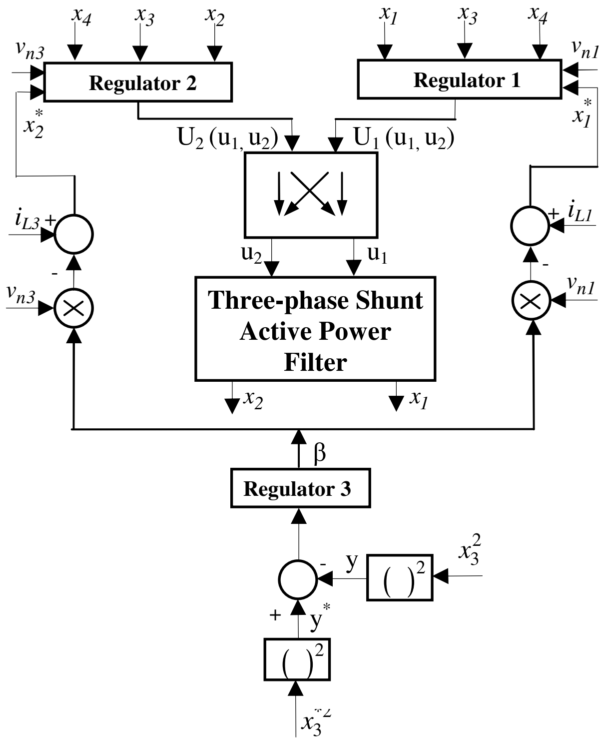

Figure 2. The structure of the controller

A. Current inner loop design

The currents i_{n1}, i_{n2} and i_{n3} delivered by the supply network must be a sinusoidal signals in phase respectively with the network voltage v_{n1}, v_{n2} and v_{n3}. Indeed the currents x_1, x_2 injected by three-phase shunt active power filter should follow the best as possible respectively the reference x_1^*, x_2^* defined by:

$$\begin{pmatrix} x_1^* = i_{L1} - \beta v_{n1} \\ x_2^* = i_{L3} - \beta v_{n3} \end{pmatrix} \tag{8}$$

Let's introduce the tracking errors on the currents x_1, x_2:

$$e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{pmatrix} \tag{9}$$

Given (7a) and (7b), time derivative of errors e_1, e_2 are written as follows:

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} -\frac{R_f}{L_f}x_1 + \frac{U_1}{L_f}x_3 + \frac{1}{6L_f}x_4 - \frac{v_{n1}}{L_f} - \dot{x}_1^* \\ -\frac{R_f}{L_f}x_2 + \frac{U_2}{L_f}x_3 + \frac{1}{6L_f}x_4 - \frac{v_{n3}}{L_f} - \dot{x}_2^* \end{pmatrix} \tag{10}$$

Since the controls U_1 and U_2 appear in (10) after a single derivation of the tracking errors, then the control laws of the currents controller will be elaborated in a single step using Lyapunov function [12].

$$V = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2\right) \quad (11)$$

Its dynamic is given by:

$$\dot{V} = \dot{e}_1 e_1 + \dot{e}_2 e_2 \quad (12)$$

The choice $\dot{V} = -c_1 e_1^2 - c_2 e_2^2$ ensuring the asymptotic stability of (10) with respect to the Lyapunov function (11) where $c_1 \succ 0$ and $c_2 \succ 0$ are a design parameters. Indeed this choice would imply:

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} -c_1 e_1 \\ -c_2 e_2 \end{pmatrix} \quad (13)$$

Comparing (13) and (10) gives the following:

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \frac{1}{x_3} \begin{pmatrix} R_f x_1 + v_{n1} - \frac{x_4}{6} + L_f \dot{x}_1^* - L_f c_1 e_1 \\ R_f x_2 + v_{n3} - \frac{x_4}{6} + L_f \dot{x}_2^* - L_f c_2 e_2 \end{pmatrix} \quad (14)$$

One deduce the expressions of effective control laws u_1 and u_2

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 4U_1 + 2U_2 \\ 2U_1 + 4U_2 \end{pmatrix} \quad (15)$$

Proposition 1: Consider the three-phase shunt APF of Fig.1 described by its averaged model (7) with the control laws (15). If $\frac{di_L}{dt}$ is measurable, the signal β and its first time-derivative are available then the inner loop system is globally asymptotically stable and its dynamic is described by (13).

B. Voltage outer loop design

The purpose of the outer loop is to generate a tuning law for the ratio β in a way that makes the output voltage x_3 regulated to a given reference value x_3^*. To this end, the relation between β and x_3 must be made clear.

Replacing in (7c) the effective laws u_1 and u_2 defined by their expressions (15) while replacing U1 and U2 by (14) an supposing negligible parasitic resistance R_f, one obtains:

$$\dot{y} = k\beta - k_1 I_1 Cos(\varphi_1) + q(t) \quad (16)$$

with

$$k = \left(k_o + \frac{2}{3} k_1 L_f \omega_n I_1 Cos\left(\frac{\varphi_1}{2} + \frac{4\pi}{3}\right) Sin\left(\frac{\varphi_1}{2}\right)\right), \; k_o = \frac{3E^2}{C_f}$$

$$k_1 = \frac{3E}{C_f}$$

$$q(t) = q_1(t) + q_2(t) + q_3(t)$$

where $q_1(t)$, $q_2(t)$ and $q_3(t)$ are given in the appendix 1.

The ratio β stands as a control input in the first-order system (16). The problem at hand is to design for β a tuning law so that the squared voltage $y = x_3^2$ tracks a given reference signal $y^* = x_3^{*2}$. Bearing in mind the fact that β and its derivative should be available (Proposition 1), a filtered PI control law is considered:

$$\beta = e_5 = \frac{b}{b+s}\left(c_3 e_3 + c_4 e_4\right) \quad (17)$$

with $\quad e_3 = y^* - y, \quad e_4 = \int_0^t e_3 d\tau$

At this point, the regulator parameters (c_3, c_4, b) are any positive real constants. The next analysis will make it clear how these should be chosen for the control objectives to be achieved.

IV. ANALYSIS OF THE CLOSED-LOOP CONTROL SYSTEM

In the following Theorem, it is shown that the control objectives are achieved (in the mean).

Theorem 1 (main result). Consider the three-phase shunt active power filter shown by Figure 1 represented by its average model (7), together with the control system consisting of the inner-loop regulator (15) and the outer-loop regulator (17). Then the closed-loop system has the following properties:

1) Let the reference signal y^* be any positive constant signal. There exists a positive real ε^* such that if $\varepsilon \le \varepsilon^*$ then, the tracking errors e_1, e_2, e_3, e_4 and the ratio β are harmonics signals that continuously depends on $\varepsilon = 1/\omega_n$, i.e. $e_1 = e_1(t,\varepsilon)$, $e_2 = e_2(t,\varepsilon)$, $e_3 = e_3(t,\varepsilon)$, $e_4 = e_4(t,\varepsilon)$, $\beta = \beta(t,\varepsilon)$

2) Furthermore, when $\varepsilon \to 0$, errors (e_1, e_2, e_3) vanishes and e_4, β converges:

$$\lim_{\varepsilon\to 0} e_1(t,\varepsilon) = 0,\ \lim_{\varepsilon\to 0} e_2(t,\varepsilon) = 0,\ \lim_{\varepsilon\to 0} e_3(t,\varepsilon) = 0,$$

$$\lim_{\varepsilon\to 0} e_4(t,\varepsilon) = \frac{k_1 I_1 Cos(\varphi_1)}{c_4 k},\ \lim_{\varepsilon\to 0} \beta(t,\varepsilon) = \frac{k_1 I_1 Cos(\varphi_1)}{k}$$

Proof. First, notice that (17) guarantees that β and its first time-derivative are available. Then, by Proposition 1, the errors e_1, e_2 undergoes (13). This together with (16) and (17) show that the augmented state vector:

$$E_r = (e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5)^T$$

undergoes the following state equation:

$$\dot{E}_r = AE_r + P(t) \tag{18}$$

where

$$A = \begin{pmatrix} -c_1 & 0 & 0 & 0 & 0 \\ 0 & -c_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & bc_3 & bc_4 & -b \end{pmatrix}$$

$$P(t) = \left(0 \quad 0 \quad k_1 I_1 Cos(\varphi_1) - q(t) \quad 0 \quad 0\right)^T$$

The stability of the above system will now be analyzed using the averaging theory. Now introduce the time-scale change $\tau = \omega_n t$. It is readily seen from (18) that $Z(\tau) \equiv E_r(\tau / \omega_n)$ undergoes the differential equation:

$$\frac{dZ(\tau)}{d\tau} = \varepsilon AZ(\tau) + \varepsilon P(\tau) \tag{19}$$

where

$$P(\tau) = \left(0 \quad 0 \quad k_1 I_1 Cos(\varphi_1) - q\left(\frac{\tau}{\omega_n}\right) \quad 0 \quad 0\right)^T$$

Now, let us introduce the average functions:

$$\bar{Z} = \lim_{\varepsilon\to 0} \frac{1}{2\pi} \int_0^{2\pi} Z(\tau) d\tau$$

$$\bar{P} = \lim_{\varepsilon\to 0} \frac{1}{2\pi} \int_0^{2\pi} P(\tau) d\tau$$

It follows from (19) that :

$$\bar{P} = \left(0 \quad 0 \quad k_1 I_1 Cos(\varphi_1) \quad 0 \quad 0\right)^T \tag{20}$$

In order to get stability results regarding the system of interest (19), it is sufficient (thanks to averaging theory) to analyze the following averaged system:

$$\dot{\bar{Z}} = \varepsilon A\bar{Z} + \varepsilon \bar{P} \tag{21}$$

To this end, notice that (21) has a unique equilibrium at:

$$Z^* = \left(0 \quad 0 \quad 0 \quad \frac{k_1 I_1 Cos(\varphi_1)}{c_4 k} \quad \frac{k_1 I_1 Cos(\varphi_1)}{k}\right)^T \tag{22}$$

On the other hand, as (21) is linear, the stability properties of its equilibrium are fully determined by the state-matrix A More specifically, the equilibrium Z^* will be globally exponentially stable if the matrix A is Hurwitz. To this end, we note that the eigen values are zeros the following characteristic polynomial:

$$det(\lambda I - A) = a_5\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$$

Where

$$a_5 = 1,\quad a_4 = b + c_1 + c_2,\quad a_3 = c_1c_2 + b(c_1 + c_2) + kbc_3$$
$$a_2 = bc_1c_2 + kbc_3(c_1 + c_2) + kbc_4,$$
$$a_1 = kbc_1c_2c_3 + kb(c_1 + c_2)c_4,\quad a_0 = kbc_1c_2c_4$$

Putting

$$b_1 = \frac{a_4a_3 - a_5a_2}{a_4},\ b_2 = \frac{a_4a_1 - a_5a_0}{a_4},\ b_3 = 0,$$
$$b_4 = \frac{b_1a_2 - a_4b_2}{b_1},\ b_5 = \frac{b_1a_0 - a_4b_3}{b_1},\ b_6 = 0,$$
$$b_7 = \frac{b_4b_2 - b_1b_5}{b_4},\ b_8 = 0,\ b_9 = 0,\ b_{10} = \frac{b_7b_5 - b_8b_4}{b_7},$$
$$b_{11} = 0,\ b_{12} = 0$$

The application of algebraic Routh criterion implies that the system is stable at the condition that:

$$a_4 > 0,\quad b_1 > 0,\quad b_4 > 0,\quad b_7 > 0,\quad b_{10} > 0$$

The equilibrium Z^* of the linear system (21) is actually globally exponentially stable. Applying Theorem 4.10 in [13], one concludes that there exists a $\varepsilon^* > 0$ such that for $\varepsilon < \varepsilon^*$, the differential equation (19) has a harmonic solution $Z = Z(t,\varepsilon)$ that continuously depends on ε. Moreover, one has $\lim_{\varepsilon\to 0} Z(t,\varepsilon) = Z^*$. This, together with (22), yields in particular that:

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$\lim_{\varepsilon\to 0} e_1(t,\varepsilon)=0$, $\lim_{\varepsilon\to 0} e_2(t,\varepsilon)=0$, $\lim_{\varepsilon\to 0} e_3(t,\varepsilon)=0$,

$$\lim_{\varepsilon\to 0} e_4(t,\varepsilon)=\frac{k_1 I_1 Cos(\varphi_1)}{c_4 k}$$

Then, using (17) one gets

$$\lim_{\varepsilon\to 0} \beta(t,\varepsilon)=\frac{k_1 I_1 Cos(\varphi_1)}{k}$$

The Theorem is thus established.

V. Numerical Simulations

In order to simulate the behavior of the three-phase shunt active power filter shown in Figure 1, the chosen nonlinear load is a three-phase full bridge rectifier which supplies an inductive load comprising a resistor R_L and an inductor L_L, the coil L_o is inserted to the input of three-phase Rectifier Bridge to limit the $\frac{di_L}{dt}$. The performances of the proposed controller are now numerically evaluated using a TPSAPF based system with the following characteristics:

TABLE I. Parameters Simulation Values

Parameters	***Symbol***	***Values***
Network	E f	$220\sqrt{2}$ V 50 Hz
Three-phase active shunt power filter	R_f L_f C_f	80 mΩ 3 mH 9000 μF
Rectifier	R_L L_L L_o	20 Ω 500 mH 5 mH
Currents regulators	$c_1=c_2$	10000
Voltage regulator	c_3 c_3 b	2×10^{-6} 6×10^{-5} 1000

The objective of the simulation is to illustrate the behavior of the controller in response to progressive changes of the voltage reference x_3^*. More specifically, the voltage reference goes from 1400V to 1500V. The performances of the controller are illustrated by Figs 3 to 10. As predicted by Theorem 1, the output voltage v_o converges, in the mean, to its reference value with a good accuracy Figure 3, a zoom is made in Figure 4 it is observed that the voltage ripples oscillates at the frequency $2\omega_n$, but their amplitude are too small compared to the average value of the signals, confirming thus Theorem 1. Fig. 5 shows that the ratio β always takes (in the mean) a constant value after transitional periods. Fig. 6 clearly shows that the network current i_{n1} is sinusoidal and in phase with the network voltages v_{n1} which shows that the power factor correction is achieved. Fig. 7 shows the control laws u_1 and u_2 in inner-loop. Fig. 8 shows the waveform of the load current i_{L1} we observe that the wave current are almost square in shape so it is rich in harmonics. The compensation current i_{f1} delivered by the three-phase shunt active power filter is shown in Figure 9. Fig. 10 shows that the network currents i_{n1} is sinusoidal and therefore testifies the good performances of the inner loop.

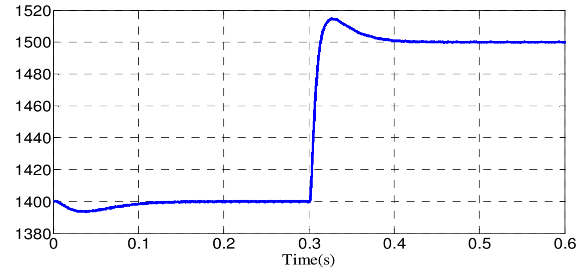

Figure 3. Output voltage v_o

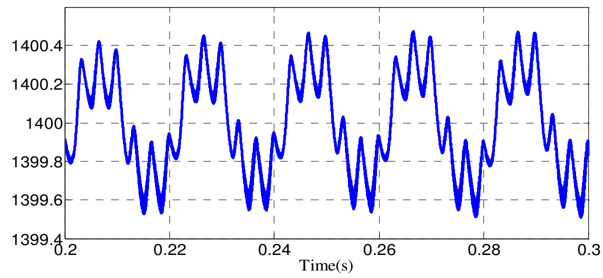

Figure 4. Ripple in the Output voltage v_o

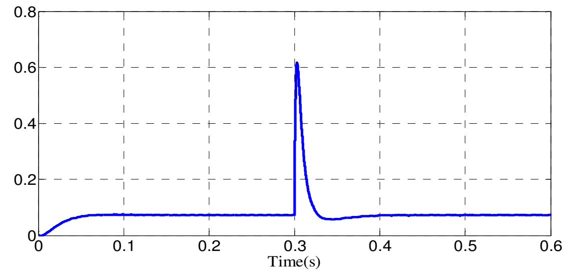

Figure 5. Control signal β

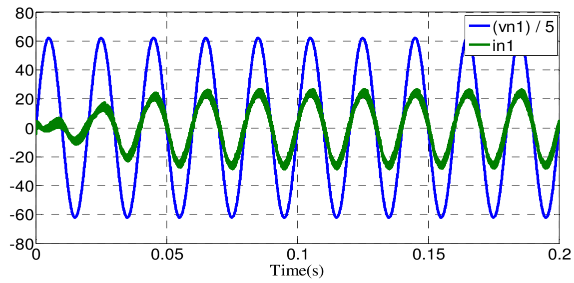

Figure 6. PFC checking: i_{n1} in phase with v_{n1}

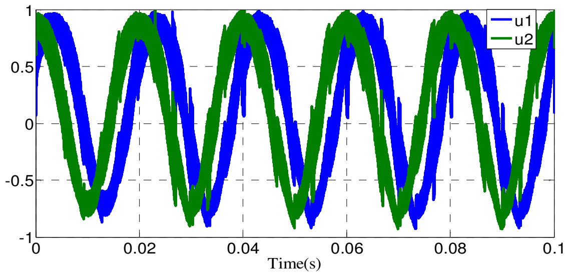

Figure 7. Inner-loop Control

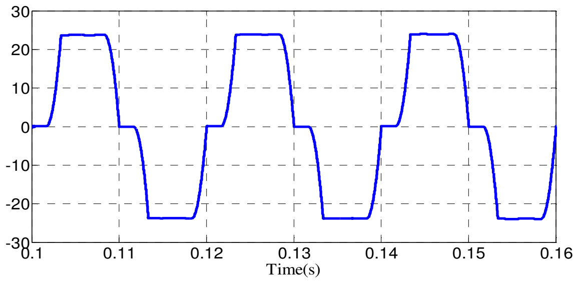

Figure 8. Load current i_{L1}

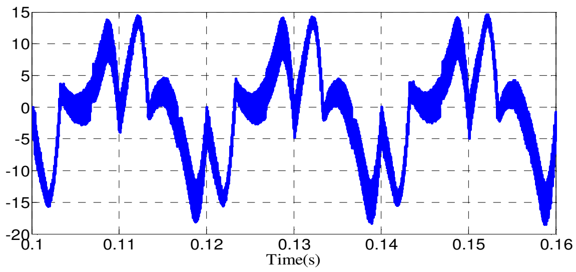

Figure 9. Compensation current i_{f1}

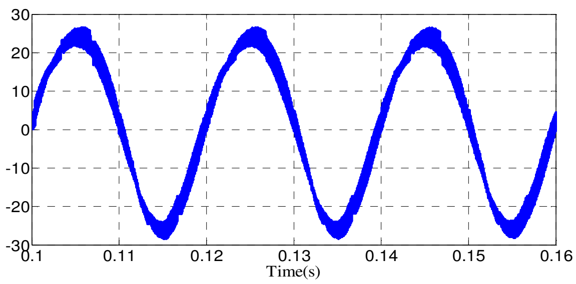

Figure 10. Network current i_{n1}

VI. CONCLUSION

In this paper, a nonlinear controller is proposed for the three-phase shunt active power filters. It aims to compensate the harmonic current and reactive power caused by non-linear loads. The problem is processed using a non-linear controller with two cascaded loops. The inner loop which is formed by two non-linear regulators is designed, using the Backstepping technique, to compensate the current harmonics. The outer loop is designed to regulate the DC bus voltage of the inverter. The simulation results show that it performs perfectly the objectives of pursuit. In addition, it is robust against changes in setpoint DC bus voltage.

APPENDIX

$$
\begin{aligned}
q_1(t) = & \frac{2L_f\beta E}{C_f} I_1\omega_n Sin\left(2\omega_n t + 2\varphi_1 - \frac{4\pi}{3}\right) \\
& + \frac{2L_f\beta}{C_f}\left(i_{L3}(t) - I_1 Sin\left(\omega_n t + \varphi_1 - \frac{4\pi}{3}\right)\right)\frac{dv_{n1}(t)}{dt} \\
& + \frac{2L_f\beta}{C_f} v_{n3}(t)\frac{d\left(i_{L1}(t) - I_1 Sin(\omega_n t + \varphi_1)\right)}{dt} \\
& - \frac{L_f\beta^2 E^2\omega_n}{C_f} Sin\left(2\omega_n t - \frac{4\pi}{3}\right) - \frac{4L_f e_2}{C_f}\frac{di_{L3}(t)}{dt} \\
& - \frac{4L_f e_2\beta}{C_f}\frac{dv_{n3}(t)}{dt} - \frac{4L_f\dot{e}_2}{C_f} i_{L3}(t) + \frac{4L_f\dot{e}_2\beta}{C_f} v_{n3}(t) \\
& - \frac{4L_f}{C_f}\frac{di_{L3}^2(t)}{dt} + \frac{8L_f\beta}{C_f}\frac{d\left(v_{n3}(t) i_{L3}(t)\right)}{dt} - \frac{16L_f}{C_f}\frac{dv_{n3}^2(t)}{dt} \\
& \frac{2L_f c_1 e_1}{C_f} i_{L3}(t) - \frac{2L_f c_1 e_1\beta}{C_f} v_{n3}(t) + \frac{4L_f c_2 e_2}{C_f} i_{L3}(t) \\
& - \frac{4L_f c_2 e_2\beta}{C_f} v_{n3}(t)
\end{aligned}
$$

$$
\begin{aligned}
q_2(t) = & \frac{2EI_1}{C_f} Cos(2\omega_n t + \varphi_1) - \frac{4}{C_f} v_{n1}(t)\left(i_{L1}(t) - I_1 Sin(\omega_n t + \varphi_1)\right) \\
& - \frac{2\beta E^2}{C_f} Cos(2\omega_n t) + \frac{2EI_1}{C_f} Cos\left(2\omega_n t + \varphi_1 - \frac{4\pi}{3}\right) \\
& - \frac{2}{C_f} v_{n3}(t)\left(i_{L1}(t) - I_1 Sin(\omega_n t + \varphi_1)\right) - \frac{\beta E^2}{C_f} Cos\left(2\omega_n t - \frac{4\pi}{3}\right) \\
& + \frac{x_4}{C_f} i_{L1}(t) - \frac{\beta x_4}{C_f} v_{n1}(t) - \frac{4L_f e_1}{C_f}\frac{di_{L1}(t)}{dt} + \frac{4L_f e_1\beta}{C_f}\frac{dv_{n1}(t)}{dt} \\
& - \frac{4L_f\dot{e}_1}{C_f} i_{L1}(t) + \frac{4L_f\dot{e}_1\beta}{C_f} v_{n1}(t) - \frac{4L_f}{C_f} i_{L1}(t)\frac{di_{L1}(t)}{dt} \\
& + \frac{4L_f\beta EI_1\omega_n}{C_f} Sin(2\omega_n t + \varphi_1) + \frac{4L_f\beta}{C_f}\left(i_{L1}(t) - I_1 Sin(\omega_n t + \varphi_1)\right)\frac{dv_{n1}(t)}{dt} \\
& + \frac{4L_f\beta}{C_f} v_{n1}(t)\frac{d\left(i_{L1}(t) - I_1 Sin(\omega_n t + \varphi_1)\right)}{dt} - \frac{4L_f\beta^2}{C_f} v_{n1}(t)\frac{dv_{n1}(t)}{dt} \\
& - \frac{2L_f e_1}{C_f}\frac{di_{L3}(t)}{dt} + \frac{2L_f\beta e_1}{C_f}\frac{dv_{n3}(t)}{dt} - \frac{2L_f\dot{e}_2}{C_f} i_{L1}(t) - \frac{2L_f\dot{e}_2\beta}{C_f} v_{n1}(t) \\
& - \frac{L_f I_1^2\omega_n}{C_f} Sin\left(2\omega_n t + 2\varphi_1 - \frac{4\pi}{3}\right) \\
& - \frac{2L_f}{C_f} i_{L1}(t)\left(i_{L1}(t) - I_1 Sin(\omega_n t + \varphi_1)\right) \\
& + \frac{2L_f\beta EI_1\omega_n}{C_f} Sin\left(2\omega_n t + \varphi_1 - \frac{4\pi}{3}\right)
\end{aligned}
$$

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$$
\begin{aligned}
q_3(t) = & + \frac{2L_f\beta}{C_f} v_{n1}(t) \frac{d\left(i_{L3}(t) - I_1 Sin\left(\omega_n t + \varphi_1 - \frac{4\pi}{3}\right)\right)}{dt} \\
& - \frac{L_f\beta^2 E^2 \omega_n}{C_f} Sin\left(2\omega_n t - \frac{4\pi}{3}\right) + \frac{2L_f c_1 e_1}{C_f} i_{L1}(t) \\
& - \frac{4L_f c_1 e_1 \beta}{C_f} v_{n1}(t) + \frac{2L_f c_2 e_2}{C_f} i_{L1}(t) - \frac{2L_f c_2 e_2 \beta}{C_f} v_{n1}(t) \\
& + \frac{EI_1}{C_f} Cos\left(2\omega_n t + \varphi_1 - \frac{4\pi}{3}\right) \\
& - \frac{2}{C_f} v_{n1}(t)\left(i_{L3}(t) - I_1 Sin\left(\omega_n t + \varphi_1 - \frac{4\pi}{3}\right)\right) \\
& - \frac{\beta E^2}{C_f} Cos\left(2\omega_n t - \frac{4\pi}{3}\right) + \frac{2EI_1}{C_f} Cos\left(2\omega_n t + \varphi_1 - \frac{8\pi}{3}\right) \\
& - \frac{4}{C_f} v_{n3}(t) \frac{d\left(i_{L3}(t) - I_1 Sin\left(\omega_n t + \varphi_1 - \frac{4\pi}{3}\right)\right)}{dt} \\
& - \frac{2\beta E^2}{C_f} Cos\left(2\omega_n t - \frac{8\pi}{3}\right) + \frac{x_4}{C_f} i_{L3}(t) - \frac{x_4\beta}{C_f} v_{n3}(t) \\
& - \frac{2L_f e_2}{C_f}\frac{di_{L1}(t)}{dt} - \frac{2L_f\beta e_2}{C_f}\frac{dv_{n1}(t)}{dt} - \frac{2L_f \dot{e}_1}{C_f} i_{L3}(t) \\
& + \frac{2L_f\beta\dot{e}_1}{C_f} v_{n3}(t) - \frac{L_f}{C_f} I_1^2 \omega_n Sin\left(2\omega_n t + 2\varphi_1 - \frac{4\pi}{3}\right) \\
& - \frac{2L_f}{C_f} i_{L3}(t) \frac{d\left(i_{L1}(t) - I_1 Sin\left(\omega_n t + \varphi_1\right)\right)}{dt} - \frac{4e_1}{C_f} v_{n1}(t) \\
& - \frac{4e_1}{C_f} v_{n3}(t) + \frac{x_4 e_1}{C_f} - \frac{4L_f e_1\dot{e}_1}{C_f} - \frac{2L_f e_1\dot{e}_2}{C_f} + \frac{4L_f c_1 e_1^2}{C_f} \\
& + \frac{2L_f c_2 e_2 e_1}{C_f} - \frac{2e_2}{C_f} v_{n1}(t) - \frac{2e_2}{C_f} v_{n3}(t) + \frac{x_4 e_2}{C_f} - \frac{2L_f e_2\dot{e}_1}{C_f} \\
& - \frac{4L_f e_2\dot{e}_2}{C_f} + \frac{2L_f c_1 e_1 e_2}{C_f} + \frac{4L_f c_2 e_2^2}{C_f}
\end{aligned}
$$

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